Spontaneous Symmetry Breaking

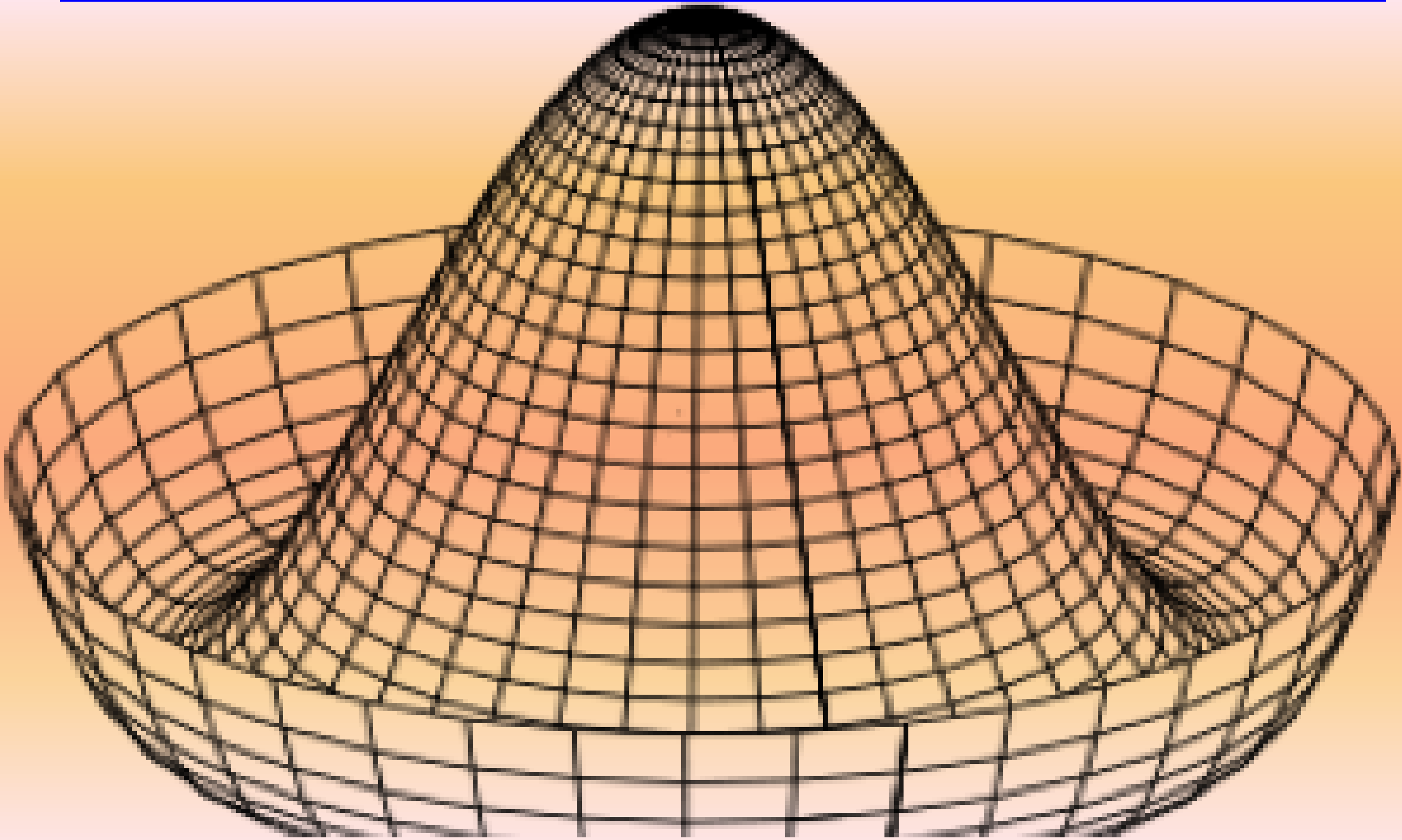
Symmetry Breaking

MASS GENERATION
WEAK BOSONS

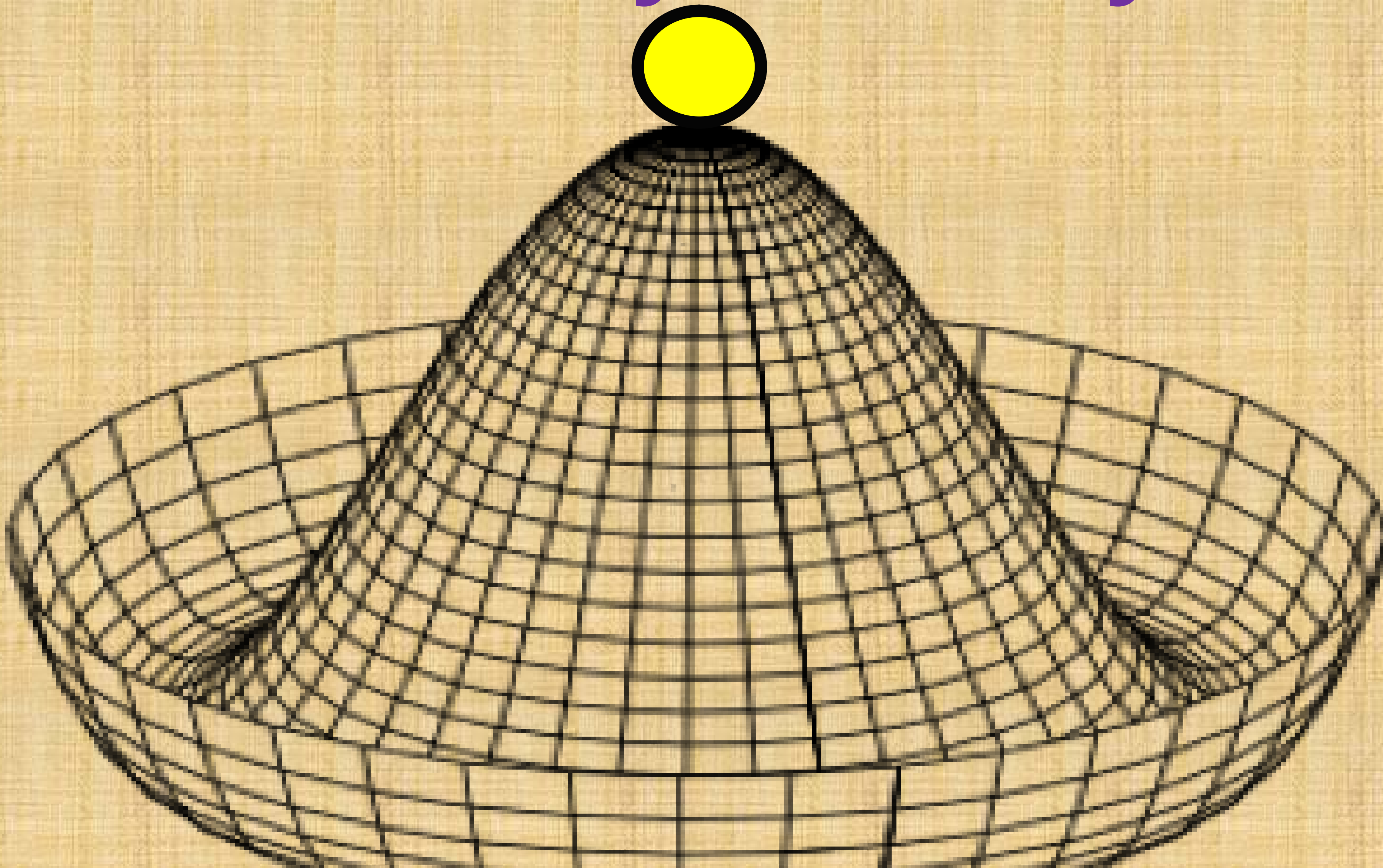
spontaneous symmetry breaking

classical mechanis

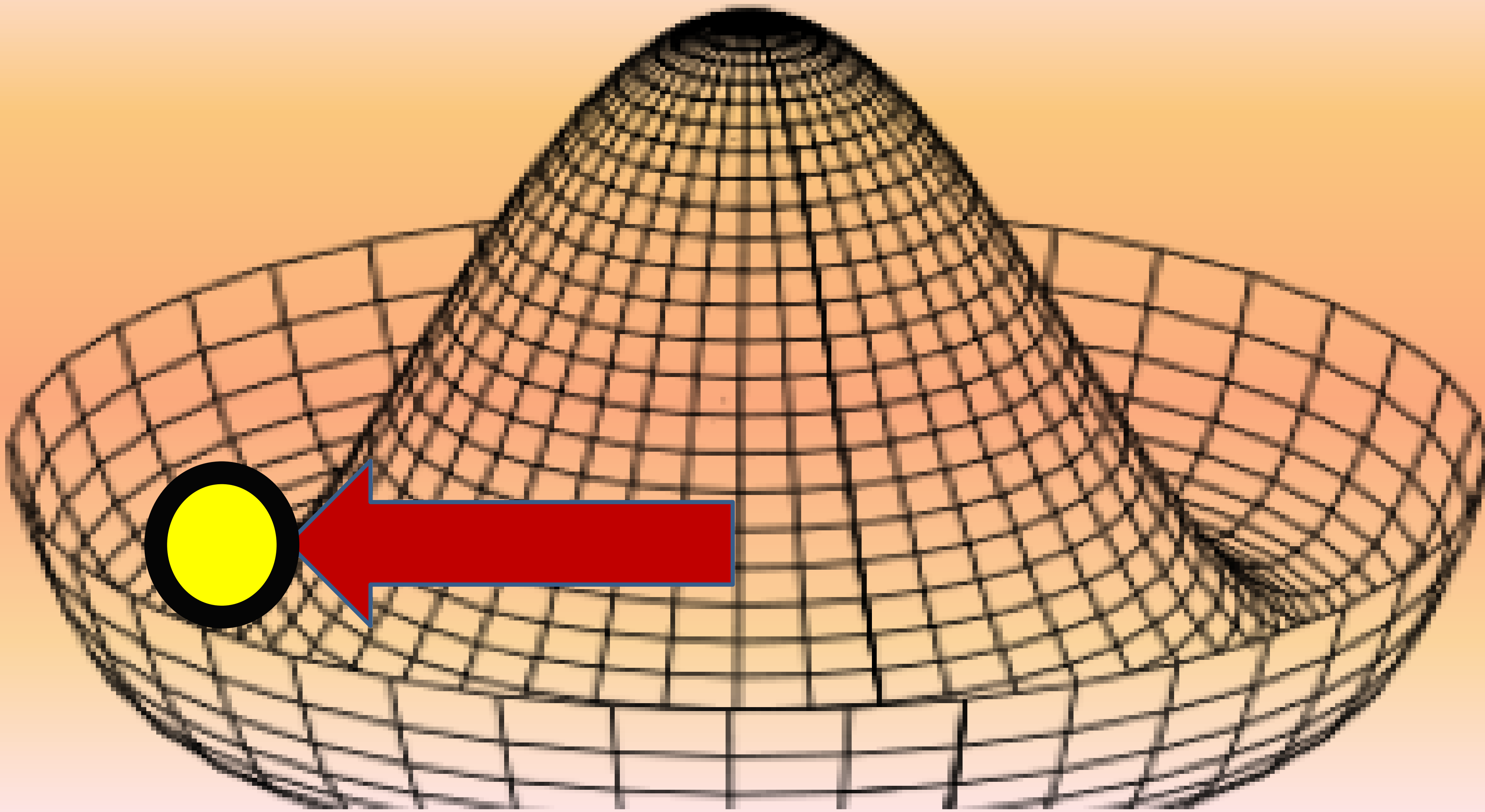
Mexican hat potential



rotation symmetry



symmetry of rotation broken
(spontaenous symmetry breaking)



spontaneous

symmetry breaking

strong interactions

isospin: generators F(i)

$$F_i = \int q^{\times} \frac{1}{2} \tau_i q d^3 x$$

$$\left[F_i, F_j\right] = i \varepsilon_{ijk} F_k$$

$$F_i = \int q^{\times} \frac{1}{2} \tau_i q d^3 x$$

$$F^5{}_i = \int q^{\times} \frac{1}{2} \tau_i \gamma_5 q d^3 x$$

quark masses zero:

vector and axialvector currents conserved

$$\partial^\mu \overline{q} \gamma_\mu q = \partial^\mu \overline{q} \gamma_\mu \gamma_5 q = 0$$

F_i, F_i^5 *conserved*

$F_- | p \rangle = | n \rangle$

$F_-^5 | p \rangle = ???$

$\left(\frac{1}{2}^- \right)$

$p : \left(\frac{1}{2}^+ \right)$

$$F_{-}|p\rangle = |n\rangle$$

$$F_{-}^{5}|p\rangle = ???$$

problem

parity doubling

m(quark) $\Rightarrow 0$ → $M_\pi \Rightarrow 0$

pion

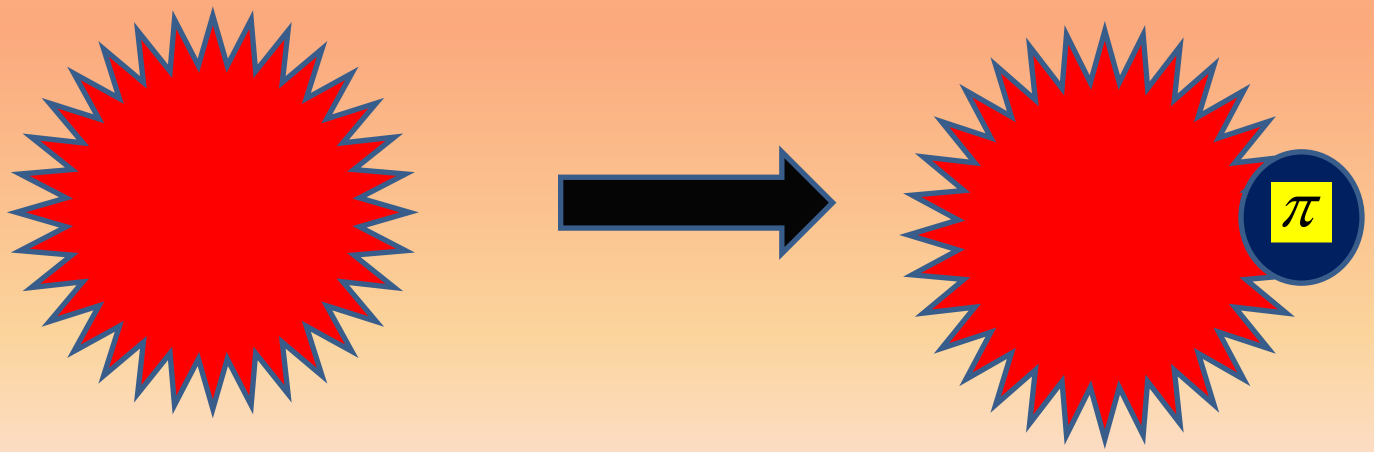
↓

MASSLESS

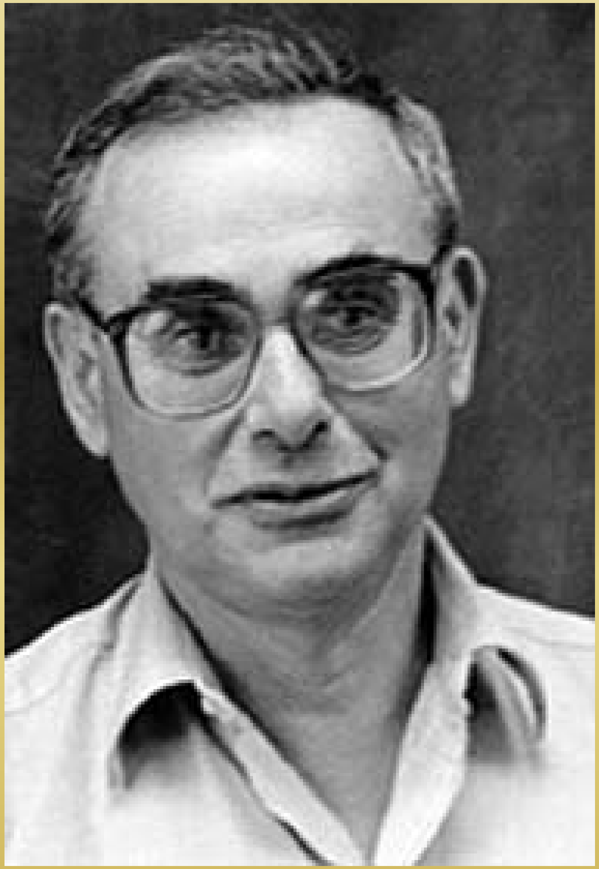
GOLDSTONE BOSON

$$F_i^5 |N\rangle = |N, \pi\rangle$$

π



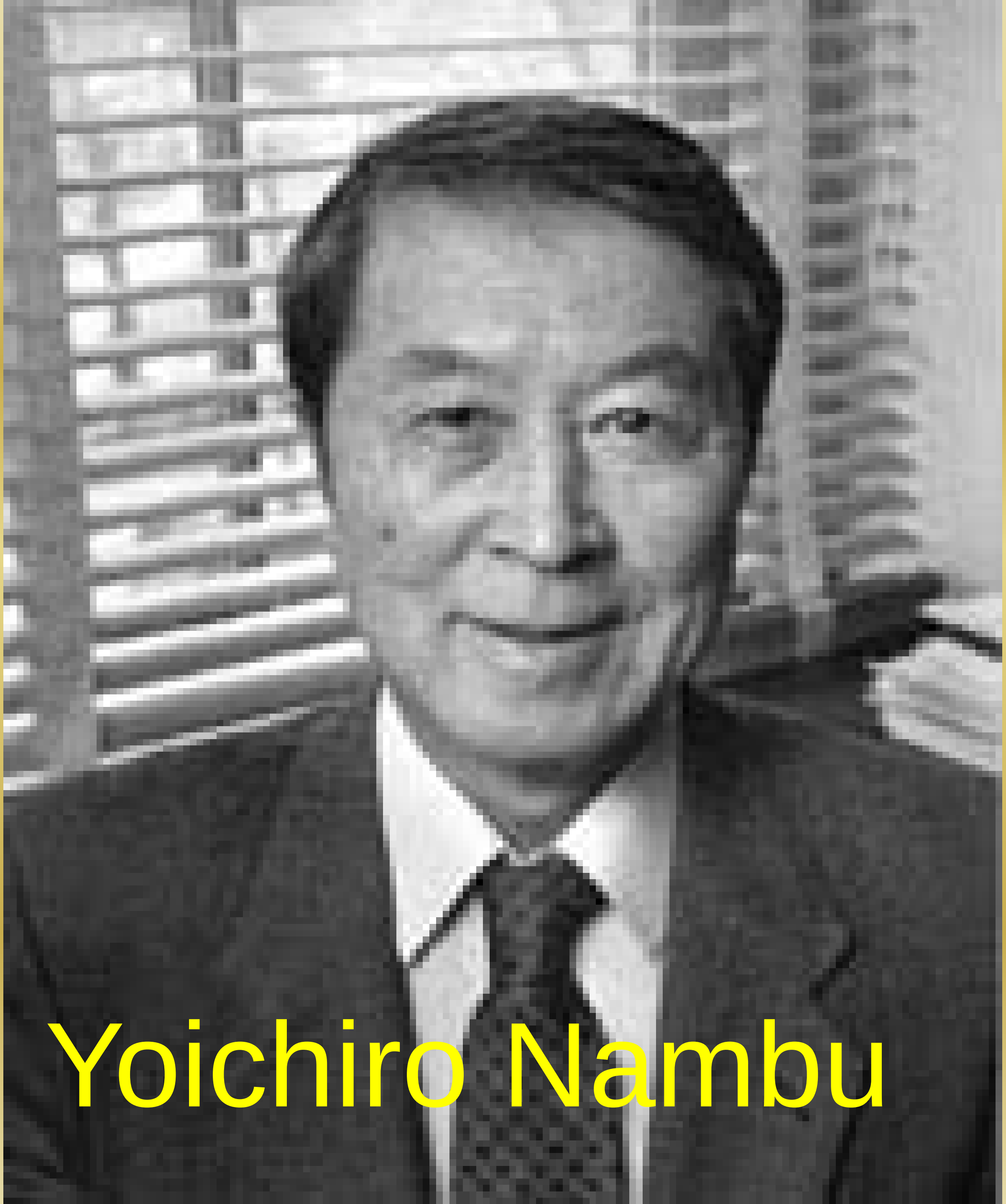
1961



J. Goldstone
MIT

Goldstone theorem

symmetry breaking
=> massless Goldstone bosons


Yoichiro Nambu

spontaneous

symmetry breaking

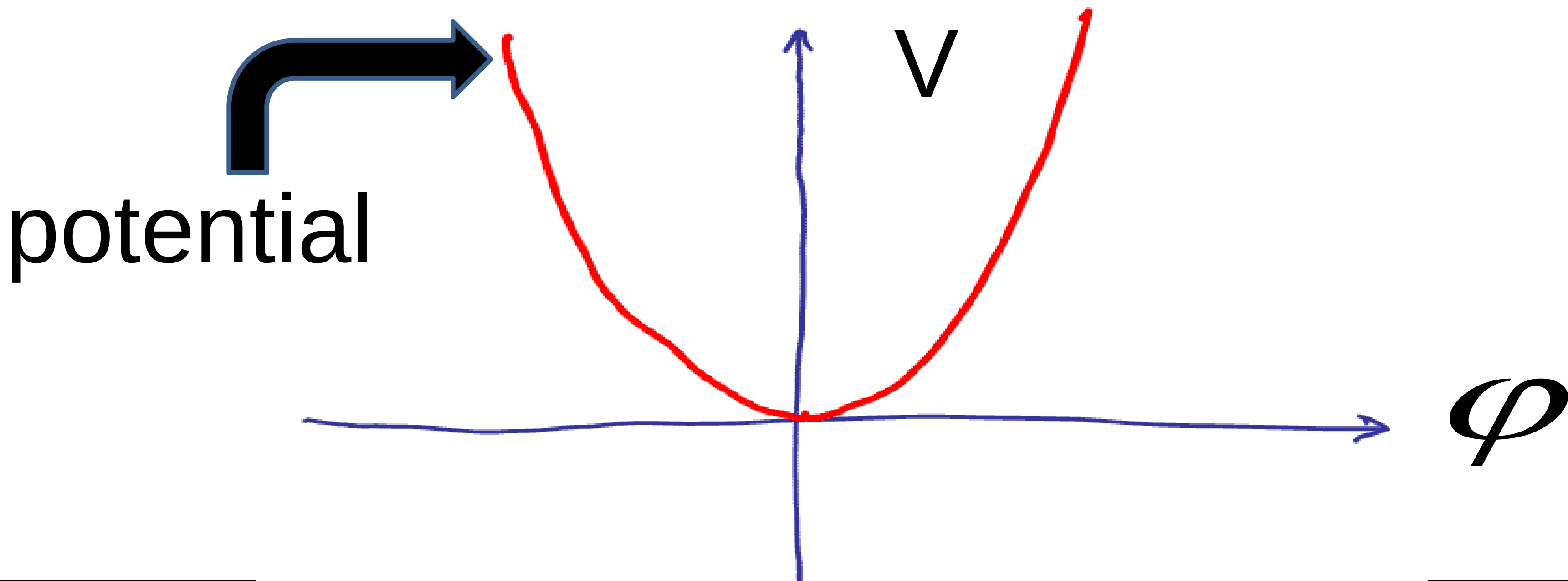
field theory

example 1: real scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}\mu^2\varphi^2 - \frac{1}{4}\lambda\varphi^4$$

$\mu^2 > 0$: scalar field – mass μ

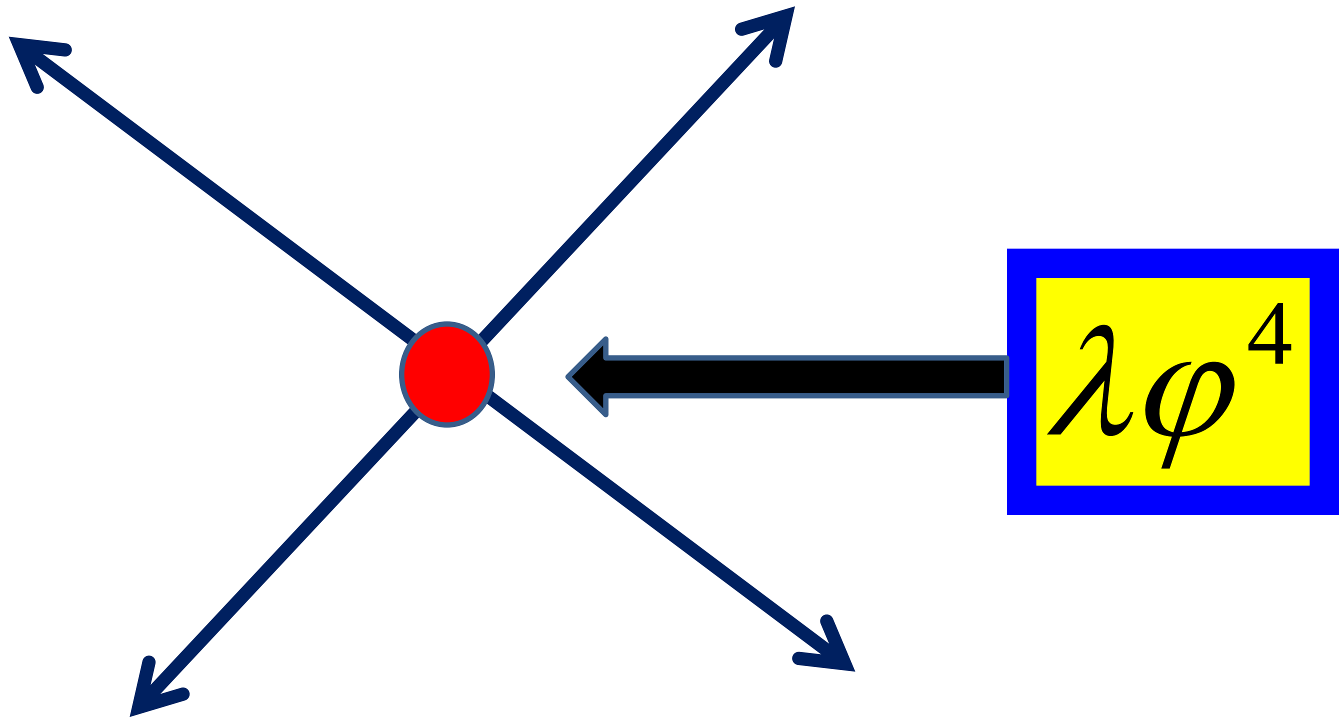
$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$$



Symmetry R

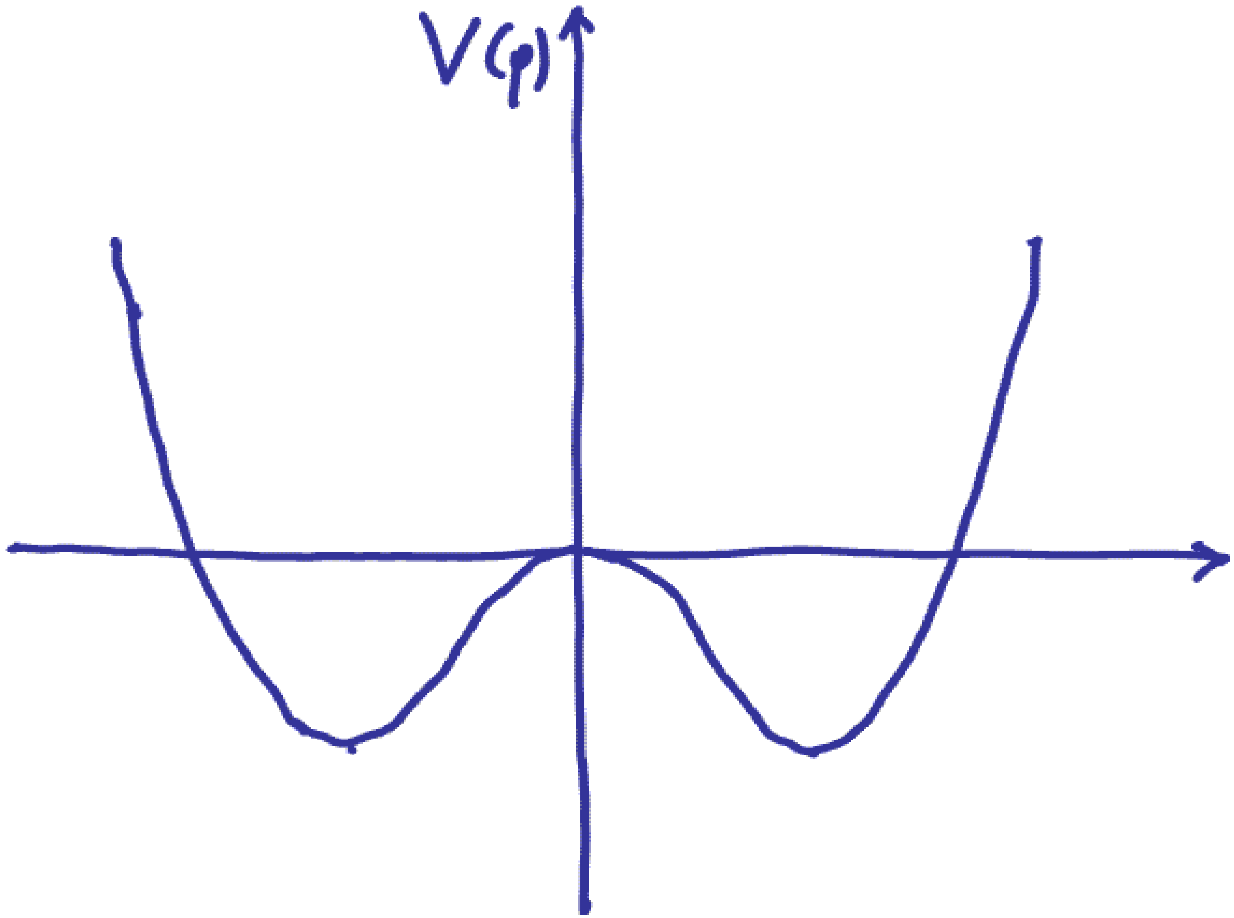
$$\varphi \xrightarrow[R]{} -\varphi$$

self - interaction:



$\longrightarrow \quad \mu^2 < 0$

$$V(\varphi) = -\frac{1}{2}|\mu^2|\varphi^2 + \frac{1}{4}\lambda\varphi^4$$



minimum of potential:

$$\varphi = \pm\sqrt{-\mu^2/\lambda}$$

ground state of field:

$$\langle 0|\varphi|0\rangle = \pm\sqrt{-\mu^2/\lambda}$$

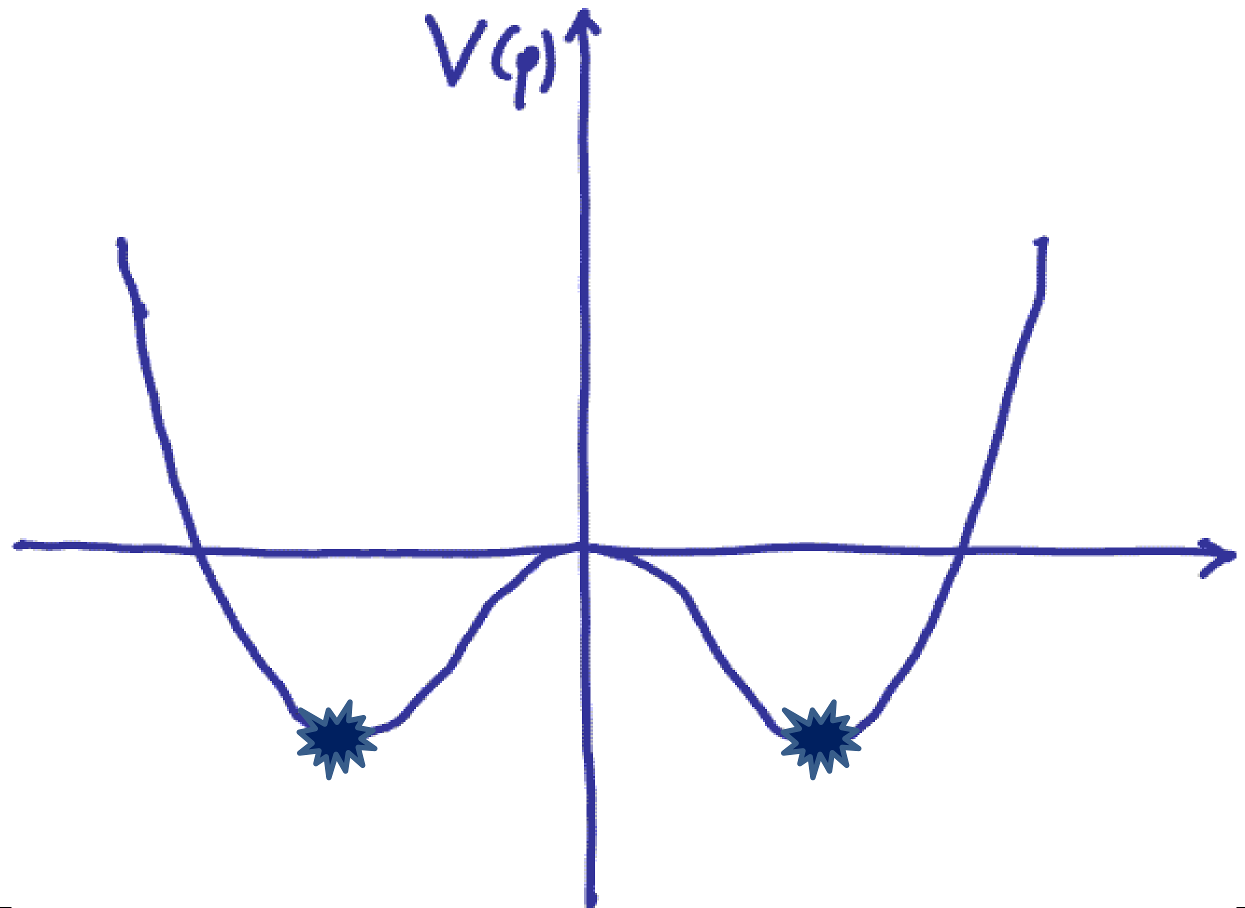
(two ground states!)

R-symmetry:

$$\varphi \xrightarrow[R]{} -\varphi$$

vacua not invariant under R!

=> 2 ground states



$$v = \langle 0 | \varphi | 0 \rangle$$

$$\varphi' = \varphi - v$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi' \partial^\mu \varphi' + \mu^2 (\varphi')^2$$

$$- \lambda v (\varphi')^3 - \frac{1}{4} \lambda (\varphi')^4 + \text{const.}$$

$\rightarrow$ scalar field, mass $\sqrt{2}\,|\mu|$

example 2:

Complex scalar field:

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$$

$$\mathcal{L} = \partial^\mu \varphi^* \partial_\mu \varphi - \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

Symmetry $U(1)$

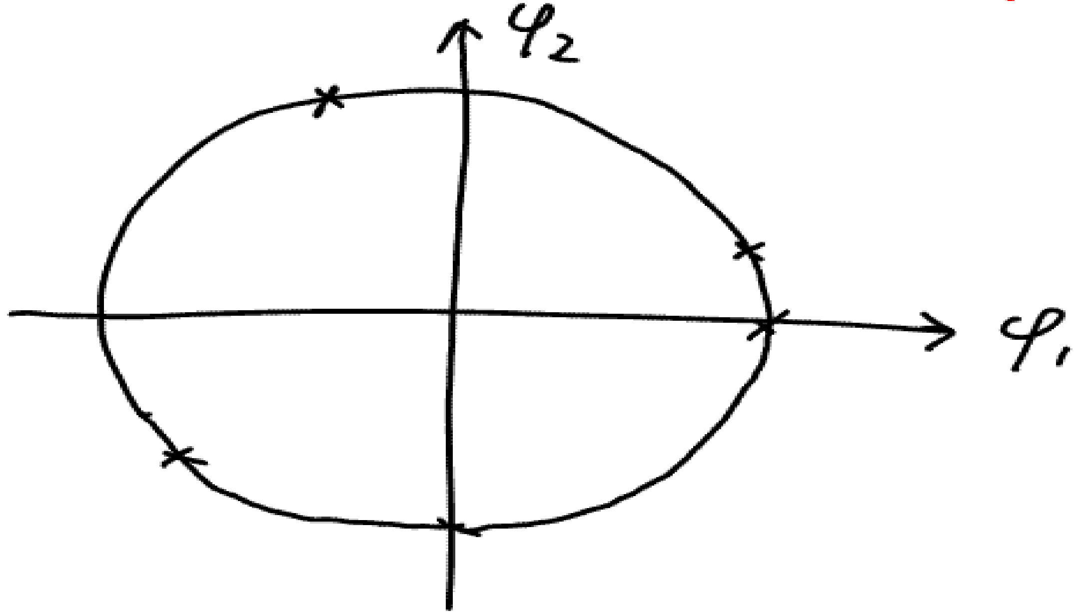
$$\varphi \longrightarrow e^{-i\theta} \varphi$$

$\mu^2 > 0$:

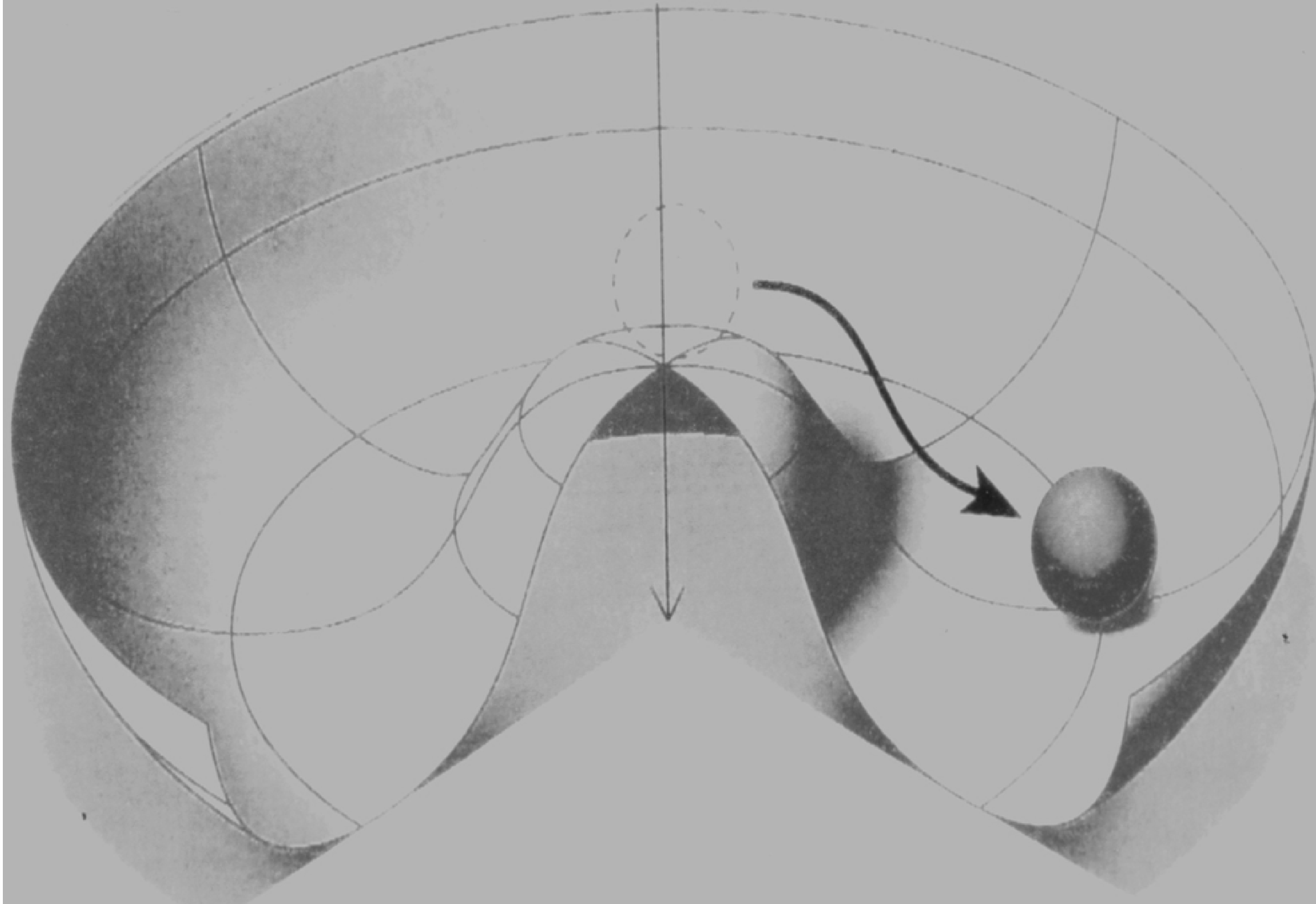
massive scalar, mass μ

$\mu^2 < 0$:

minimum of potential at circle - radius $\sqrt{-\mu^2/\lambda}$



∞ vacuo



Choose vacuum

$$\langle 0|\varphi_1|0\rangle = \sqrt{-\mu^2/\lambda}$$

$$\langle 0|\varphi_2|0\rangle = 0$$

$\rightarrow$ φ_1: particle $\sim$ mass $\sqrt{2}|\mu|$

φ_2: massless

($\rightarrow$ Goldstone boson $\sim$ spontaneous symmetry breaking spontaneous mass generation

gauge theories

U(1)

Spontaneous mass generation:

scalar field, interacting with a gauge field

$$\mathcal{L} = (D^\mu \varphi)^* (D_\mu \varphi) - \mu^2 (\varphi^* \varphi) - \lambda (\varphi^* \varphi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

$$D_\mu = \partial_\mu + i g A_\mu$$

$\mu^2 > 0$:

massive scalar

$\updownarrow$

massless gauge field

$\mu^2 < 0$:

spontaneous breaking

of symmetry

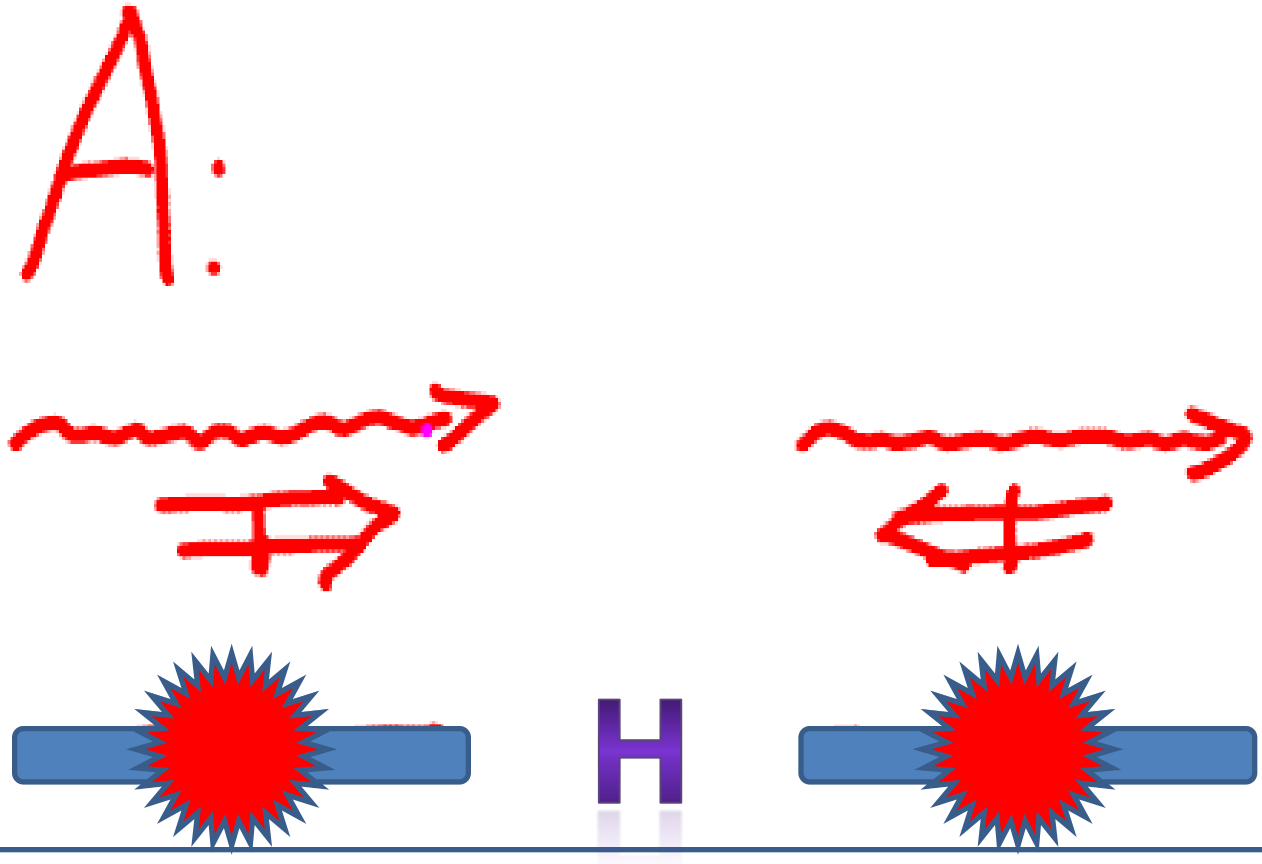
$\mu^2 < 0$: $\langle 0|\varphi_1|0\rangle = \sqrt{\frac{-\mu^2}{\lambda}}$

$\varphi_1 = \varphi_1' + v \qquad v = \sqrt{\frac{-\mu^2}{\lambda}}$

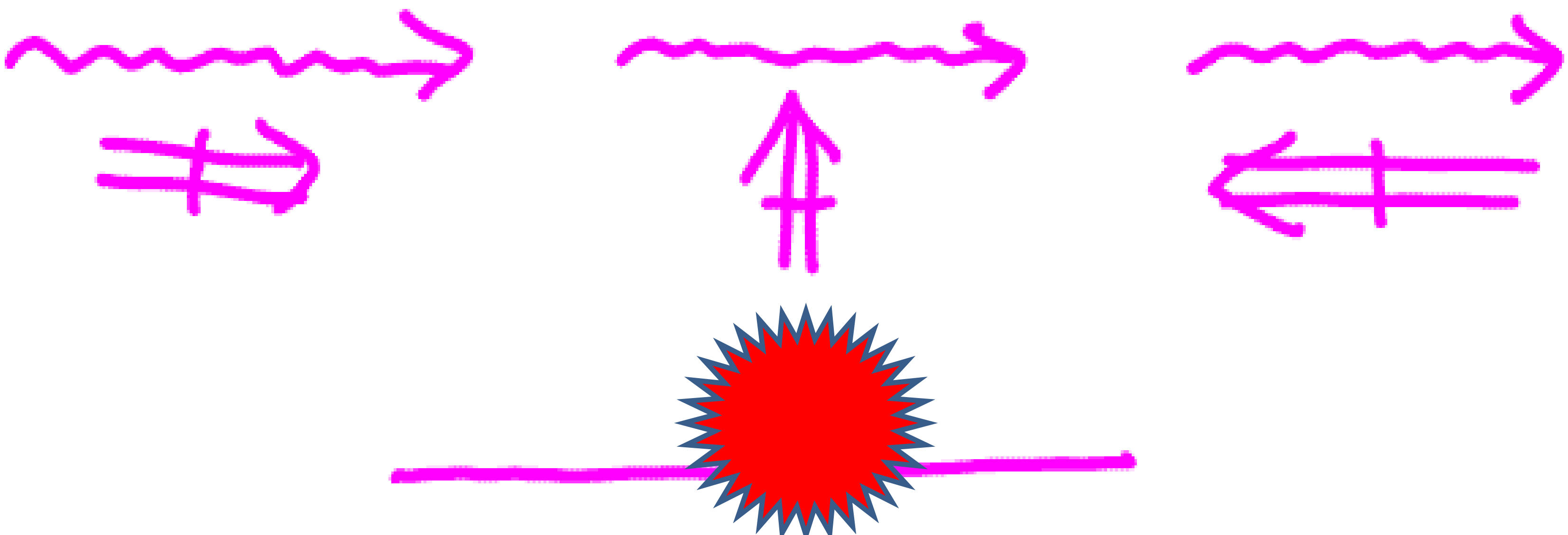
$\rightarrow$ mass term for gauge field

$$\frac{1}{2} g^2 v^2 (A_\mu A^\mu)$$

exact symmetry



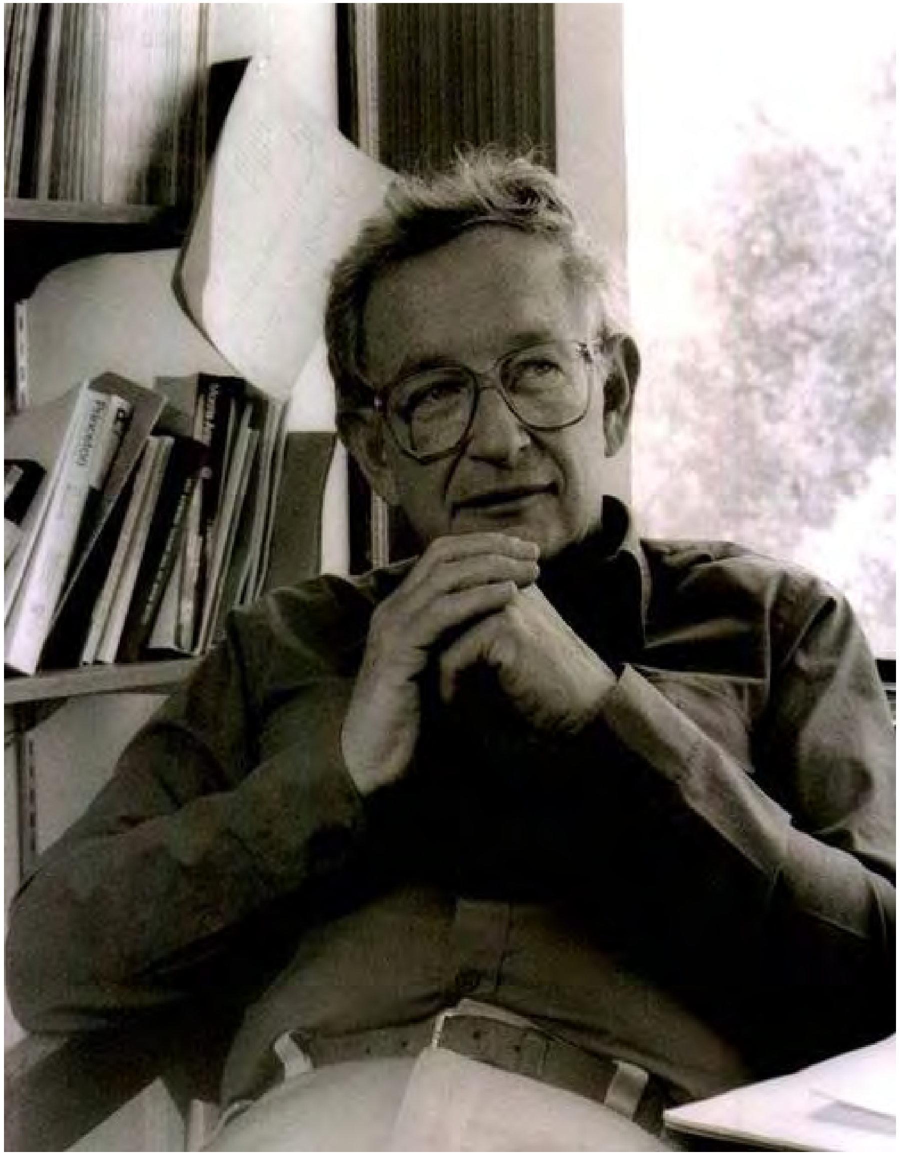
B: broken symmetry



one scalar „Goldstone – boson“ absorbed by the gauge boson

↓

mass generation



Phil Anderson

Peter Higgs


Kibble Guralnik Hagen Englert Brout

SU(2)

scalars:
doublet

$$\mathcal{L} = -\frac{1}{4} G^{i}_{\mu\nu} G^{\mu\nu}_{i}$$

$$+\left(\partial^{\mu}\varphi + i\frac{g}{2}\tau^{i} B^{\mu i}\varphi\right)^{*}\left(\ . \quad . \quad . \ \right)$$

$$-\mu^{2}\varphi^{*}\varphi \quad - \lambda\left(\varphi^{*}\varphi\right)^{2}$$

φ : $SU(2)$ doublet

$\mu^2 < 0$:

$$\langle 0|\varphi|0\rangle \longrightarrow \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$v = \sqrt{-\mu^2/\lambda}$$

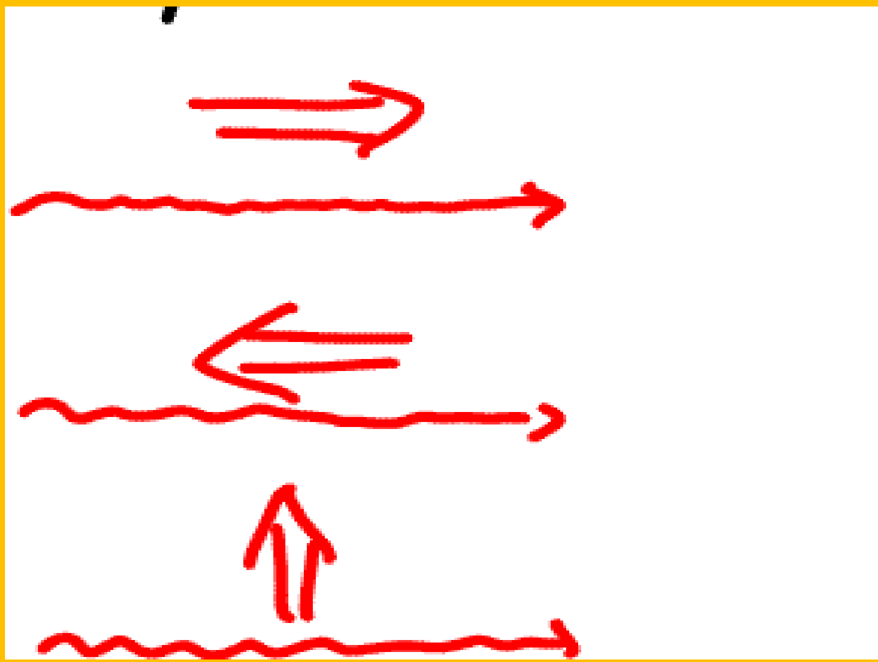
$$\longrightarrow \left(\frac{g^2}{8}\right) v^2 \left[(B_\mu^1)^2 + (B_\mu^2)^2 + (B_\mu^3)^2\right]$$

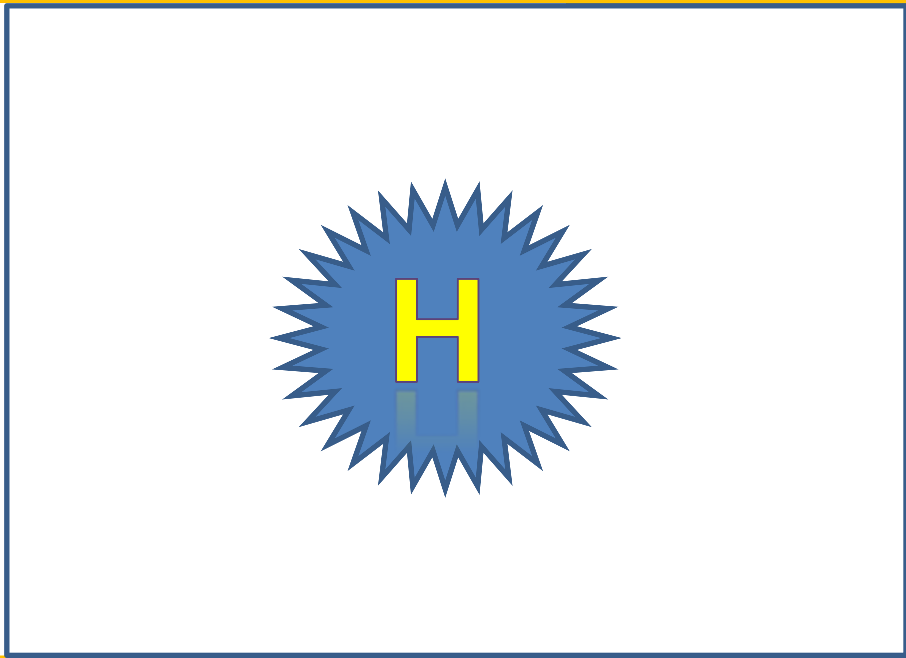
3 degenerate gauge bosons
(massive)

degrees of freedom before symmetry breaking:

3 massless gauge bosons : 4 massive scalars

after symmetry breaking





3 scalars absorbed by 3 gauge bosons

one scalar remains

(Higgs boson)

SU(2)

scalars:
triplet

$$\mathcal{L} = -\frac{1}{4} G^{i}_{\mu\nu} G^{\mu\nu}_{i}$$

$$+ \frac{1}{2} \left(\partial_\mu \varphi_i - g \varepsilon_{ijk} B^\mu_j \varphi_k \right) \left(\ldots \right)$$

$$- \frac{1}{2} \mu^2 (\varphi_i \varphi_i) - \frac{1}{4} \lambda (\varphi_i \cdot \varphi_i)^2$$

$$\mu^2 < 0 \longrightarrow$$

$$\langle 0 | \varphi | 0 \rangle \longrightarrow \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

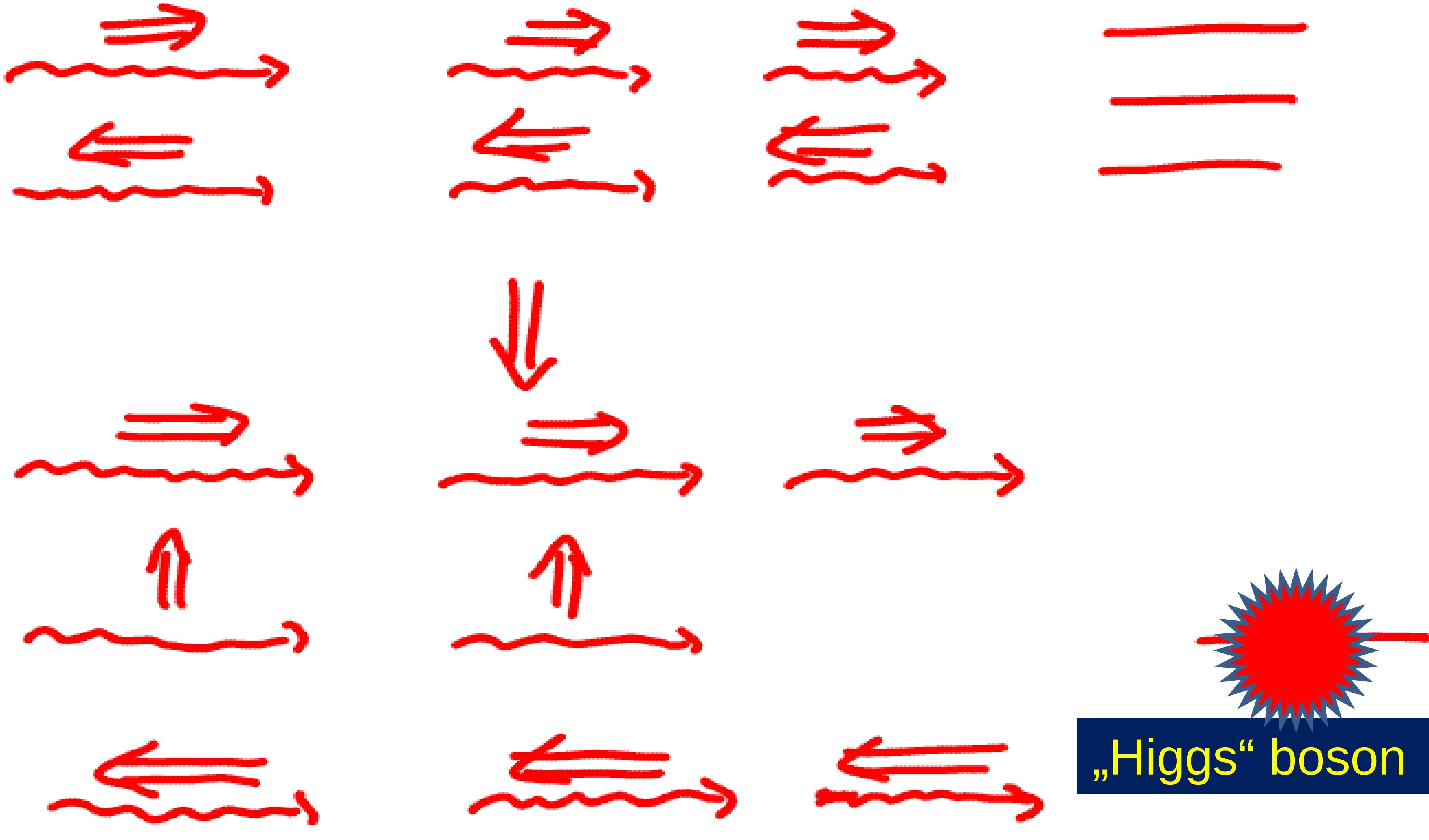
$\longrightarrow$ mass term:

$$\frac{1}{2} g^2 v^2 \left((B_\mu^1)^2 + (B_\mu^2)^2 \right)$$

$\rightsquigarrow$ 2 massive gauge bosons

1 gauge boson massless

degrees of freedom:



minimal electroweak theory

gauge group: SU(2) x U(1)

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad e^-_R$$

$$Q = T_3 + \frac{1}{2} Y$$

$$Y(e^-_L) = Y(\nu_e) = -1$$

$$Y(e^-_R) = -2$$

3 gauge bosons

$A_\mu^i \longrightarrow T_i$

1 gauge boson

$B_\mu \longrightarrow Y$

$$\mathcal{L}^{g.b.} = -\frac{1}{4} F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

$$\mathcal{L}^{fermion} = \overline{L}\, i \left(\partial\!\!\!/ + i \frac{g}{2} \tau_i A\!\!\!/_i - \frac{g'}{2} B\!\!\!/ \right) L$$

$$+ \overline{R}\, i \left(\partial\!\!\!/ - i g' B\!\!\!/ \right) R$$

$$\mathcal{L}^{scalar} =$$

$$\left(\partial^\mu \varphi^* - \frac{i}{2} g' B^\mu \varphi^* - \frac{i}{2} g A_\mu^i \tau_i \varphi^*\right)$$

$$\cdot \left(\;\ldots\; \right) - V(\varphi^* \varphi)$$

potential:

$$V = \mu^2 (\varphi^* \varphi) + \lambda (\varphi^* \varphi)^2$$

$$\mu^2 < 0$$

$$\longrightarrow \langle 0|\varphi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$v = \sqrt{-\mu^2/\lambda}$$

symmetry breaking:

SU(2) and U(1) broken
electric charge unbroken

$$M_{W^\pm} = \frac{1}{2} g \cdot v$$

$$Z = \frac{-g A^3 + g' B}{\sqrt{g^2 + g'^2}}$$

$$A = \frac{g B_\mu + g' A_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

$$M_A = 0$$

weak angle θ_W:

$$\tan\theta_W = \frac{g'}{g}$$

$$\leadsto \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

$$\longrightarrow \quad v \cong 246 \text{ GeV}$$

LEP

mass of Higgs boson

> 114 GeV

LHC

???